Assignment 2

This homework is due Friday Feb 5.

There are total 35 points in this assignment. 31 points is considered 100%. If you go over 31 points, you will get over 100% for this homework (but not over 115%) and it will count towards your course grade.

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper *and give credit to your collaborators in your pledge.* Your solutions should exhibit your work and reasoning. Bare answers will not earn you much.

This assignment covers Sections 1.3–1.4 of Textbook.

(1) [10pt] Evaluate the following.

(a) $\left|\frac{4-3i}{2-i}\right|$.
(b) $|(1+i)^{2016}|$.
(c) $|z-2|^2$, where $z = x+iy$.
(d) $\operatorname{Arg}(1-i)$.
(e) $\operatorname{Arg}\left(\frac{2}{1-i\sqrt{3}}\right)$.
(f) $\operatorname{Arg}((1+i\sqrt{3})(1+i))$.
(g) $\operatorname{Arg}(\cos 2 - i\sin 2)$.
(h) $\operatorname{Arg}(-2-9i)$.

(2) [10pt] Represent the following complex numbers in polar form (either with sin and cos, or with $e^{i\theta}$, whichever you like more). (*Hint:* Find absolute value first, factor it out, then find appropriate angle θ)

(a) -2015.
(b) $\cos\theta$.
(c) $6i-6$.
(d) $-2\sqrt{3}-2i$.
(e) $\sin\theta - i\cos\theta$.
(f) $-e^{i\theta}$.
(g) $12-5i$ (express argument in terms of inverse trig functions).
(h) $-7-4i$ (express argument in terms of inverse trig functions).

(3) [5pt] Compute using polar form.
(a) $(i-\sqrt{3})^3(1+i\sqrt{3})^2$.
(b) $\frac{(1+i)^{2017}}{(1-i)^{2015}}$.
(c) $(\sin 3 - i\cos 3)^{-2}(-\cos 1 - i\sin 1)$.

(4) [10pt] Sketch and name the sets of points determined by the following relations.
(a) $|z| = 3$.
(b) $|z+1-2i| = 2$.
(*Hint:* Remember that $|z-z_0|$ is *distance* from z_0 to z.)
(c) $\operatorname{Re}(z+1) = 0$.
(d) $|z+3i| \le 2$.
(e) $\operatorname{Im}(z-i) > 6$.

(5) [5pt, optional]
(a) Let $z \neq 1$. Show that $1 + z + z^2 + \ldots + z^n = \frac{1-z^{n+1}}{1-z}$.
(b) Compare Re of LHS and RHS in the above equality to show that

$$1 + \cos\theta + \cos 2\theta + \ldots + \cos n\theta = \frac{1}{2} + \frac{\sin\left((n+\frac{1}{2})\theta\right)}{2\sin\frac{\theta}{2}} \text{ for } 0 < \theta < 2\pi.$$